Bostwick Degree and Rheological Properties: an Up-to-date Viewpoint

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Abstract:
The correlation between the Bostwick degree and the static rheological properties of yield stress food fuids is first revisited and then reformulated in this work. The role of the yield stress in the free surface flow of the Bostwick test is studied using dimensional analysis. Results from experiments on 48 different samples of yield stress fluids are considered and included to check the adequacy of the proposed correlation. Asymptotic dynamic behaviour is also presented and discussed as a mechanism of complete self similarity with respect of the dimensionless time. This approach would seem to support the opinions in favor of the yield stress as a key parameter, and thus offers an interesting new viewpoint useful to both future experiments on the Bostwick test and studies of 'dam-break' like dynamics.

Zusammenfassung:
In dieser Arbeit wird die Wechselbeziehung zwischen dem Bostwick-Grad und den statischen rheologischen Eigenschaften von flüssigen Nahrungsmitteln behandelt und neu formuliert. Die Rolle der Fliessspannung im freien Oberflächenfluss des Bostwick-Tests wurde mit Hilfe der Dimensionsanalyse untersucht. Resultate von Experimenten mit 48 verschiedenen Proben von Flüssigkeiten mit einer Fliessspannung wurden in diese Studie einbezogen, um die Richtigkeit der vorgeschlagenen Wechselbeziehung zu überprüfen. Zusätzlich wird eine Analyse des asymptotischen dynamischen Verhaltens vorgestellt und diskutiert. Dieser Ansatz scheint die Theorie der Fliessspannung als Hauptparameter zu unterstützen, und bietet folglich eine neue und interessante Sichtweise, welche sowohl für zukünftige Experimente mit dem Bostwick-Test als auch für Untersuchungen von "dammbrechungsähnlicher" Dynamik nützlich ist.

Résumé:
Dans ce travail la corrélation entre le degré Bostwick et les propriétés rhéologiques statiques des fluides alimentaires avec contrainte seuil est d'abord revisitée et ensuite reformulée. Le rôle de la contrainte seuil dans l'écoulement de Bostwick est étudié en utilisant l'analyse dimensionnelle. Les résultats des expériences entreprises sur 48 échantillons différents de fluides avec contrainte seuil sont aussi considérés et inclus pour vérifier l'adéquation de la corrélation proposée. Une analyse du comportement dynamique asymptotique est également présentée et discutée. Cette approche semblerait soutenir les avis en faveur de la contrainte seuil comme paramètre principal, et offre ainsi un nouveau point de vue, intéressant et utile aux futures expériences sur l'essai de Bostwick de même que sur les études sur la dynamique de rupture.

Key words: Bostwick consistometer, yield stress, viscoplastic fluids, food rheology, dimensional analysis, fruit purees

1 INTRODUCTION

Many products from the food industry show a non-Newtonian rheological behavior of the shear thinning type with the presence of yield stress [1 - 3]. From an engineering viewpoint, the yield value can be considered a characteristic of several food fluids, typically of those whose raw products have a high content of fibers (e.g. fruits and vegetables). The static rheological characteristics of these products have been extensively described by means of two or three parameter models, such as the Ostwald-deWaele or the Herschel-Bulkley [1, 4, 5]. Among the rheological properties time dependency is considered to be relevant for particular applications, i.e. in cases that imply either relevant material degradation or structure formation [5, 6]. On the contrary, the variability of viscosity on shear, i.e. namely the apparent viscosity, is always a key issue. Its importance ranges from speculative to more practical interests, an immediate example being fluid characterization, which is fundamental to many industrial processes. For a given flow condition, a way of quantifying the apparent viscosity is to define it as $\eta = \tau/\dot{\gamma}$, where τ and $\dot{\gamma}$ are the current shear stress and strain, respectively. However, to calculate the apparent viscosity there is the objective difficulty that both the stress and strain need to be known. For a certain number of applications (an example is the control of the food quality for common commercial purposes) the possibility of summarizing the variability of viscosity into a unique mean value has therefore long been studied in the past. The story seems to have began around 1938 when E.P. Bostwick (quoted in [7]) of the U.S. Department of Agriculture developed an instrument to evaluate what he called the 'consistency' of food products. Under his viewpoint, such a quantity was a way to embed under a unique parameter the ensemble of those rheological characteristics responsible for the non-Newtonian viscous behavior. A very simple technique resulted, i.e. the so-called Bostwick consistency or Bostwick test. This test, which details will be discussed in the next sections, consists of measuring the distance covered by a given sample of fluid over a flat slot in a conventional time interval and at constant temperature. Although this simple experiment has been so far widely used by industries, there are still many uncertainties that make the comparison of data coming from different sources difficult. Reasons for such embarrassing situation mainly concern with the incomplete understanding of the role that some variables have in the dynamics and with the widespread properties of the products to test. The case of the yield stress is an example: although it is known that such fluids cannot spread indefinitely [2], the role of the yield stress in explaining the Bostwick measure has long been debated in the past.

An analysis of the Bostwick experiment by tracking the front evolution at four different time steps of yield stress fluids is thus presented in this work. The spanned time interval includes the instant at which the Bostwick measure is usually made, nonetheless allows to better explore the whole process after the rapid initial transition has depleted. The aim is to furnish some hints on the role that the yield value has in the late stage of the dynamics, so to help filling some gap still left open from previous researches. A mathematical correlation and a graphical diagram both useful for practical purposes are thus obtained and commented. The hope is that these results can both stimulate useful discussions and suggest how future steps toward an improvement of the Bostwick technique should be addressed.

The paper is organized as follows: apart from this introductory section, the next one deals with a general review of both the instrument characteristics and the most relevant results obtained in the past. The approach using dimensional analysis is then proposed in Section 3. The results of 48 samples of fruit fluids are presented in Section 4 in order to validate the proposed correlation, whereas discussion and conclusions are left in Section 5.

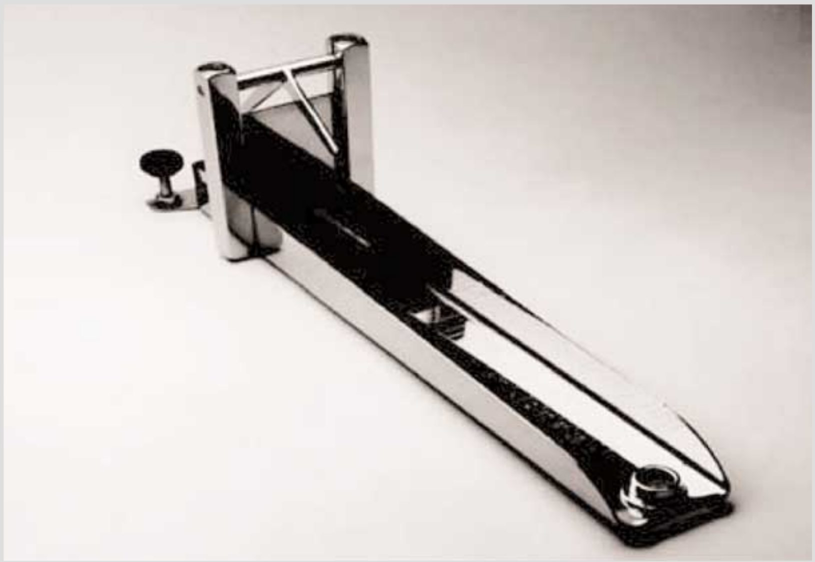


Figure 1: The narrow Bostwick consistometer.

2 APPARATUS AND HISTORY: A BRIEF REVIEW

2.1 THE BOSTWICK CONSISTOMETER

Figure 1 illustrates the Bostwick consistometer. The same device has been used for the experiments carried out in this work. Commercial consistometers are mainly distinguishable for the ratio between width b and height H of the sample reservoir (b/H = 1.3 for the narrow consistometer and b/H = 6.6 for the wide one). The Bostwick consistometer is designed to sit at a specified small angle, which is usually neglected. One ensures that the sloped flowing lane is at the right position by adjusting a series of screws until the levelling bubble on the front of the instrument is centered. The sample reservoir of the narrow instrument has a capacity $V = b^2H$ of approximately 100 ml (i.e. $b \approx 0.05$ m and $H \approx 0.04$m) and connects the flow lane through a sluice gate. The flow lane is basically a rectangular aisle which surface roughness is usually very small and can be assumed as being smooth. Its length can range up from 0.24 to 0.50 m, depending on the consistency of the products being tested.

A conceptually similar to dam-break flow occurs as soon as the sluice gate is released and the material filling the reservoir starts to flow along the aisle. The reading (in centimeters) of the distance covered after a conventional time (usually 30 s for pureed products or 10 s for tomato sauces) and averaged over three consecutive tests is the Bostwick consistency, hereby referred to as B_{30}.

2.2 WHAT IS KNOWN FROM THE PAST

The experiment with the Bostwick consistometer is empirically funded. The distance covered by the sample within the established time interval gives a rough idea of the mean resistance (perhaps an equivalent mean viscosity) that the fluid experiences against deformation in this particular type of gravity-driven flow. Thanks to its simplicity, the Bostwick device is often used to check and accordingly adjust the consistency of manufacturing products via a feedback. For this reason, the possibility of implementing such a technique directly in or on-line has long been studied. The first interesting device was proposed by D. Eolkin in 1958 [7] as a continuous recording consistometer that he himself called the plastometer. Appositely designed for non-Newtonian fluids, such an instrument was able to correlate the Bostwick consistency versus the apparent viscosity. The reason for this good correlation, as pointed out by Eolkin himself, was the ability of the plastometer to catch the role of the structural component (i.e. something responsible for the yield stress and, in turn for the apparent viscosity) and to relating it to the Bostwick measure. The ability of the plastometer was then proved with the paradoxical case of a sufficiently concentrated sugar syrup (which still shows a Newtonian viscosity) and a yield stress puree. As expected, the Bostwick consistometer and the plastometer came up to two completely different conclusions [7]: the first apparatus stated that these products were exactly the same, while the plastometer actually captured the difference. Twenty years later, Rao and Bourne [8] revisited the plastometer to investigate in more detail the reason for the excellent correlation that it showed with the Bostwick reading. They suggested that this fact was more imputable to the apparent viscosity and not to the yield stress property of the fluid. However, as also pointed out by the authors themselves, the weak point of the analysis was the estimate of the yield stress with an empirical model instead of measuring it. After such first debates, other authors came up to either coherent or contrasting conclusions, thus resetting the problem to a sort of basic starting point. For instance, Vercruysse and Steffe [9] accurately analyzed baby food products and found a poor correlation between Bostwick and apparent viscosity. Singh et al. [10] suggested the use of the Brix degree (i.e. the sugar content) for in-line continuous measurements. Barringer et al. [11] studied the correlation of Bostwick data with measured differential pressure in pipe flow; Trifiro et al. [12] used an oscillatory viscosimeter and Alamprese et al. [13] correlated the Bostwick with the parameters of an Ostwald-de Waele rheological model.

Not surprisingly all such results showed both agreement and discrepancies among each other, thus exactly reflecting the deep meaning of Eolkin's paradox. In other words, the fact that fluids show an internal structure responsible for the yield stress or, more generally for an apparent viscosity, cannot be distinguished by the Bostwick measure. This major concern proves that such a measure does not have a true physical meaning and does not allows for a univocal identification of the rheological properties of the fluid being tested. However, whether a first general classification of the fluid can be made (e.g. with or without yield), then some better information can be obtained even using such a simple instrument. To this end, by an undoubtedly different direction moved McKarthy and Seymour [14, 15]. They approached the problem from an analytical point of view conscious of the rheological nature of the fluid being modelled.

In their approach, the phenomenon was simplified to a one-dimensional gravity current which can be described analytically and allowed to obtain an elegant solution that is valid for both Newtonian [14] and non-Newtonian [15] fluids of the Ostwald-deWaele type. In particular, the linearized form of de Saint Venant equations was corrected in the resistive term in order to account for the non-Newtonian effects. The solution was then obtained by means of a similar solution technique (see, for example the book by Barenblatt [16]), and the corresponding results allowed to show the physics underlying the correlation between Bostwick degree, fluid density and apparent viscosity. Such rather interesting conclusions, although are still not able to take the effects of the yield stress into account, suggest the need of this further step.

3 DIMENSIONAL ANALYSIS OF THE BOSTWICK TEST

3.1 FORMULATION FOR YIELD STRESS FLUIDS

The longitudinal displacement of the front at several time intervals can be related to the rheological properties of viscoplastic fluids (such structured food fluids, for instance) by means of dimensional analysis. The fluid is assumed to behave as a purely viscous time independent Herschel-Bulkley material, which constitutive equation in the limits of a one dimensional approximation is $\tau = \tau_O + K\dot{\gamma}^n \cdot \tau_O$ is the yield stress, while K and n are the 'consistency' and flow indexes, respectively (see, for example [3] for a general overview). Some consideration is further made to keep the number of variables affordable from a practical point of view. Surface tension for example will not be considered here accordingly to other authors [17 - 20], albeit its importance in the asymptotic flow should be better investigated. Elastic and time dependent effects are also neglected assuming that the products show neither pronounced elastic components, nor relevant structure degradation under shears of limited duration and intensity [5]. Finally, as already said, the lane slope is also negligible i.e. the Bostwick apparatus is assumed as horizontal [14, 15].

Assuming the length L covered by the front along the centreline is the governed quantity, then the governing variables can be reasonably identified with the mass of the sample M, gravity g, rheological parameters (τ_O, K, n), initial height of the fluid at rest H and the time t. The physical link can therefore be stated as

$$L = f(M, g, \tau_o, K, t, H; n) \tag{1}$$

where the flow index n appears as a simple parameter being itself dimensionless. Notice that the mass M instead of the fluid density ρ has been voluntarily chosen to describe the motion of a finite volume $V = M/\rho$ of fluid. This choice comes from the need of describing the motion of a finite amount of product and not the evolution of a continuously feeded stream. Both the information can be taken thus into account within a unique quantity while maintaining in the relationship the characteristic dimension H of the device. The Buckingham or Π-theorem [16] states that Eq. 1 can be rewritten in dimensionless form provided that a representative set of dimensionally independent quantities is considered. For example, the triplet M, g, H is adequate and can be adopted to make Eq. 1 dimensionless

$$\Pi_1 = \Phi(\Pi_2, \Pi_3, \Pi_4, n) \tag{2}$$

where the dimensionless groups are

$$\Pi_1 = \frac{L}{H} \qquad \Pi_2 = \frac{\tau_o H^2}{Mg} \tag{3}$$

$$\Pi_3 = t\sqrt{\frac{g}{H}} \qquad \Pi_4 = \frac{Kg^{\frac{n-2}{2}}}{MH^{\frac{n-4}{2}}} \tag{4}$$

Notice that by choosing these quantities the plot of the curves at parameter Π_3 = *cost* is thus simplified. The dimensionless group Π_1 represents a simple scaling of the travelled distance, whereas the meaning of Π_2 is more deeper. Once this group is rewritten by explicitly using the fluid density ρ and the volume of the sample $V = b^2H$,

one gets $\Pi_2 = \alpha \frac{\tau_o}{\rho g H}$.

Apart from the constant α, which depends on the device geometry, such a quantity is the Bingham number. This number compares the yield stress magnitude with the initial hydrostatic pressure level created by gravity. Although a similar quantity was already found to be useful in other previous works [17, 20, 21], the one introduced here also accounts for the amount of material (i.e. the mass M) that is involved in the process. To this regards, some recent works have proved the importance of the total fluid mass [17, 19], the initial shape of the pile and the channel geometry [19] in determining the stoppage of the flow (see, [2] for a general overview).

3.2 REDUCTION

Equation 2 describes a rather complex hyper-surface difficult to fit to experimental data unless a large number of samples measured with high accuracy is available. The dimension of the problem needs thus to be further reduced by doing some reasonable considerations. Since the process being studied occurs after a rapid initial transitory, by that time the mass deforms at relatively low shear rates. Thus, the rheological model shows that the role of the yield stress in the phenomenon prevails on that of the rheological parameters *K* and *n*. This fact seems to be reasonable since the yield stress contribute is more important to the fluid stoppage than the viscous terms. Moreover, the yield plays an important role on the apparent viscosity since the flow starts, slowly making the flow history, which in principle can influence the front evolution, to cover only a secondary role. In the absence of strong tixotropic effects, the final shape of the fluid is in facts dictated by the equilibrium condition where all the internal stresses equal the yield [17, 22; see also next subsection]. The apparent viscosity of a Herschel-Bulkley material is the sum of two components, one due to the yield value τ_O and the other to the viscous terms *K* and *n* of the constitutive law, i.e. $\eta = \eta_{\tau O} + \eta_{K,n}$. Therefore, from the definition of apparent viscosity one gets

$$\eta = \frac{\tau_o}{\gamma} + K\gamma^{n-1} \tag{5}$$

The less pseudoplastic the fluid, the higher the contribute given by the yield stress is when compared with that of the consistency index at low shear rates. In particular, such a condition means that $\eta_{K,n} << \eta_{\tau O}$, i.e. in dimensional form this occurs when

$$\Pi_4 \gamma^n << \Pi_2 \tag{6}$$

In the previous relation the term $\gamma = \gamma\sqrt{H/g}$ can be only calculated provided the shear rate at the wall is known. However, this delicate point is not fundamental to the present theory as long as one focus at the long term of the process. Therefore, for a first calculation the inequality stated in Eq. 6 can be estimated starting from the average velocity and a representative height of the yielded region [18 - 20]. When the hypothesis expressed by Eq. 6 holds, then the two rheological indexes of consistency *K* and flow *n* can be removed from the functional link Eq. 1, and thus Eq. 2 reduces to

$$\Pi_1 = \Psi(\Pi_2, \Pi_3) \tag{7}$$

The form of the unknown dimensionless function Ψ can be sought now from experimental data.

3.3 ASYMPTOTIC CONDITION

Giving sufficient time, the mass spreading will end up with the arrest of the flow [2, 19]. In the limit of the approximation made, and for geometrically similar devices, the final extent is only dependent on the fluid properties. The standstill condition is characterized by a resulting free surface profile that is no longer horizontal, which relevance has been proved to be important also for practical purposes, and allowed some techniques, e.g., the slump test to be developed [17, 22]. This characteristic behaviour suggests that the more the group Π_3 increases, the less its importance in the dynamics. At the same time, since a finite distance will be approached at rest i.e. a finite limit for Π_1 does exist, then

$$\lim_{\Pi_3 \to \infty} \Pi_1 = \lim_{\Pi_3 \to \infty} \Psi(\Pi_2, \Pi_3) = \Psi(\Pi_2) \tag{8}$$

This condition is consistent with the ones already found in literature [17, 19, 22]. In particular, it shows the existence of a complete self similarity [16] for the variable Π_3 in the dynamics, and allows to reduce the link between the active variables to only two groups.

4 EXPERIMENTAL VERIFICATION

4.1 MATERIALS AND METHODS

The experimental part has dealt with some typical fluid products derived from fruits, i.e. the so-called fruit purees both raw and diluted with water. Such products contain a conspicuous percentage of pulp and show a common characteristic, i.e. the yield stress. In order to obtain a greater variety of products showing less viscosi-

Product	Dil	L (5s) [m]	L (15s) [m]	L (30s) [m]	L (60s) [m]	τ_0 [Pa]	K [kg m^{-1} s^{n-2}]	n	ρ [kg/m^{-3}]
Apricot 1	0%	0.042	0.045	0.046	0.048	21.70	4.35	0.44	1037.0
	3%	0.046	0.048	0.049	0.051	19.61	2.56	0.52	1035.9
	5%	0.049	0.052	0.055	0.055	15.47	3.53	0.47	1035.1
	7%	0.061	0.065	0.066	0.067	14.22	1.89	0.53	1034.4
	10%	0.064	0.069	0.070	0.072	9.17	2.26	0.50	1033.3
	15%	0.069	0.073	0.074	0.075	8.98	1.77	0.53	1031.4
Apricot 2	0%	0.148	0.150	0.150	0.150	3.22	0.31	0.69	1037.0
	12%	0.182	0.183	0.184	0.184	2.25	0.20	0.73	1032.6
	17%	0.211	0.212	0.212	0.212	1.50	0.12	0.78	1030.7
	20%	0.218	0.220	0.220	0.220	1.20	0.10	0.81	1029.6
Pear 1	0%	0.040	0.043	0.044	0.044	20.21	7.94	0.38	1060.0
	3%	0.043	0.046	0.047	0.049	17.59	5.93	0.42	1058.2
	5%	0.043	0.045	0.048	0.049	16.84	5.15	0.44	1057.0
	7%	0.047	0.050	0.052	0.054	16.09	4.74	0.45	1055.8
	10%	0.053	0.055	0.058	0.059	12.72	3.47	0.49	1054.0
	15%	0.055	0.058	0.059	0.060	12.35	3.13	0.48	1051.0
Pear 2	0%	0.117	0.123	0.127	0.133	4.24	1.52	0.51	1060.0
	12%	0.143	0.149	0.156	0.162	2.50	0.95	0.54	1052.8
	17%	0.165	0.173	0.180	0.189	1.72	0.67	0.53	1049.8
	20%	0.181	0.188	0.195	0.203	1.50	0.40	0.65	1048.0
Apple 1	0%	0.037	0.040	0.043	0.044	19.46	10.68	0.31	1060.0
	3%	0.041	0.045	0.046	0.049	15.91	8.50	0.33	1058.2
	5%	0.048	0.050	0.052	0.054	10.98	8.35	0.35	1057.0
	7%	0.049	0.052	0.054	0.057	9.73	6.89	0.37	1055.8
	10%	0.054	0.057	0.059	0.062	9.98	5.55	0.39	1054.0
	15%	0.061	0.064	0.067	0.069	9.48	3.71	0.43	1051.0
Apple 2	0%	0.040	0.044	0.047	0.051	19.69	6.62	0.38	1060.0
	12%	0.057	0.061	0.065	0.068	12.72	2.36	0.50	1052.8
	17%	0.068	0.073	0.075	0.077	11.73	2.01	0.51	1049.8
	20%	0.073	0.078	0.081	0.084	8.73	1.76	0.53	1048.0
Strawberry	0%	0.057	0.072	0.081	0.091	9.21	9.28	0.31	1015.0
	5%	0.065	0.082	0.092	0.102	7.73	9.14	0.30	1014.2
	10%	0.072	0.088	0.099	0.110	7.49	7.78	0.32	1013.5
	12%	0.077	0.096	0.106	0.117	5.74	6.58	0.32	1013.2
	15%	0.082	0.100	0.111	0.124	5.61	6.89	0.32	1012.7
	17%	0.087	0.105	0.118	0.131	5.24	6.41	0.34	1012.4
	20%	0.089	0.110	0.123	0.135	4.24	8.86	0.33	1012.0
Peach	0%	0.154	0.156	0.156	0.158	2.25	0.39	0.64	1015.0
	3%	0.156	0.158	0.160	0.163	2.25	0.12	0.75	1014.5
	5%	0.155	0.158	0.160	0.163	1.50	0.32	0.66	1014.2
	7%	0.173	0.174	0.175	0.177	1.50	0.23	0.71	1013.9
	10%	0.183	0.185	0.187	0.189	1.31	0.21	0.72	1013.5
	15%	0.183	0.185	0.187	0.191	1.12	0.11	0.81	1012.7
Banana	0%	0.075	0.083	0.089	0.097	9.23	0.35	0.63	1011.0
	10%	0.097	0.100	0.110	0.117	6.48	0.51	0.67	1009.9
	20%	0.119	0.126	0.132	0.140	5.24	0.24	0.74	1008.8
	30%	0.157	0.162	0.168	0.177	2.75	0.24	0.72	1007.7
	40%	0.180	0.186	0.187	0.201	1.90	0.13	0.78	1006.6

Table 1: Rheological and physical data of the raw and diluted products at the temperature of 25°C.

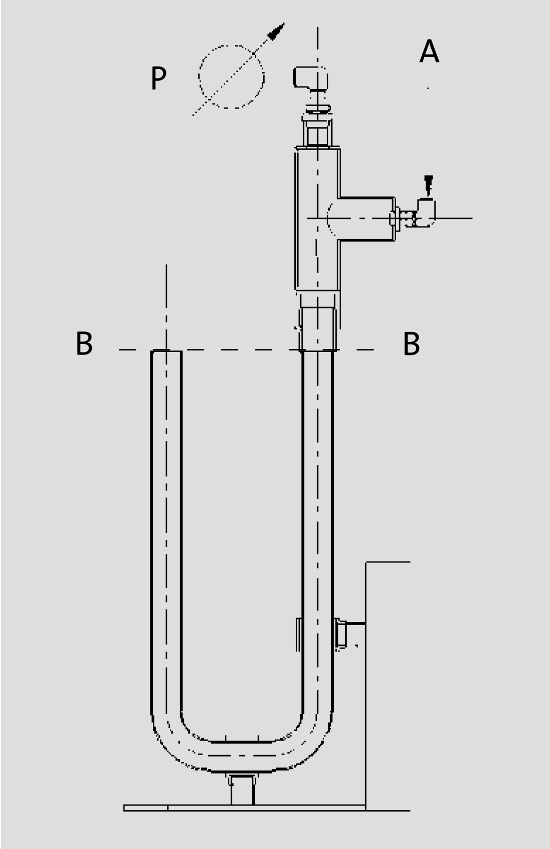


Figure 2 (left above): The U-tube device used to estimate the yield stress. The fluid meniscus originally stands at the same level (B-B); air is inflated in (A) and the pressure transducer (P) is connected to a data acqui sition system (LabView).

ty, they were diluted with different percentage in volume of water. Forty-eight fluid samples were thus prepared, each of which showing different rheological and physical properties (Table 1). Among such products only the strawberry puree showed significant elastic properties. Sample of this products were however included in order to confer generality to the the work.

Yield stress

The yield value τ_0 was assessed by using the direct technique of the U-tube, which furnishes an accuracy that matches with the industrial needs of economy, rapidity and sufficient reliability. This was preferred to other indirect meth-

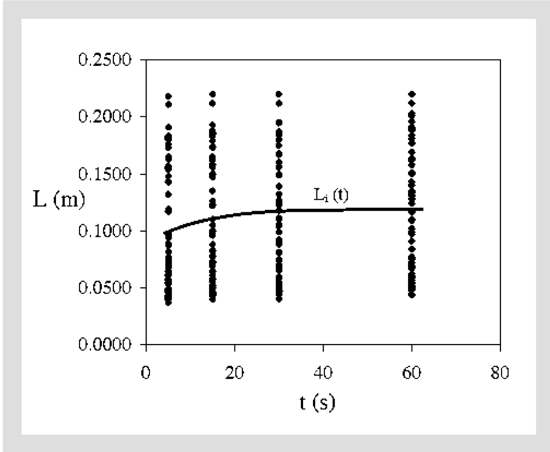


Figure 4 (right above): Plot of the front evolution for all the measured data.

ods - such as those based on the extrapolation of rheological models to make the measure objective and, at the same time, independent of the choice of the adopted constitutive model and of errors due to the fitting procedure. The U-tube was also preferred to the alternative slump test direct method (see, for example [17, 22] in order to avoid problems of serum separation; for the materials hereafter explored such effects can be relevant and might therefore influence the results. The U-tube in this sense prevents such a separation in that the whole sample is confined. This technique assesses an engineering value of the yield stress starting from the measure of the minimum pressure gradient that is required to trigger the flow into a U-curved pipe. Figure 2 shows the experimental device that was accurately designed by the author and used for the measurements. It is a circular steel pipe with an internal diameter D_1 = 0.025 m and a net length L_1 = 0.835 m. At one end of the instrument, the pressure is gradually augmented by inflating air very slowly and intermittently, so to generating a 'quasi-static' process. Differential pressure Δp at the two ends of the tube was measured by using a membrane pressure transduc-

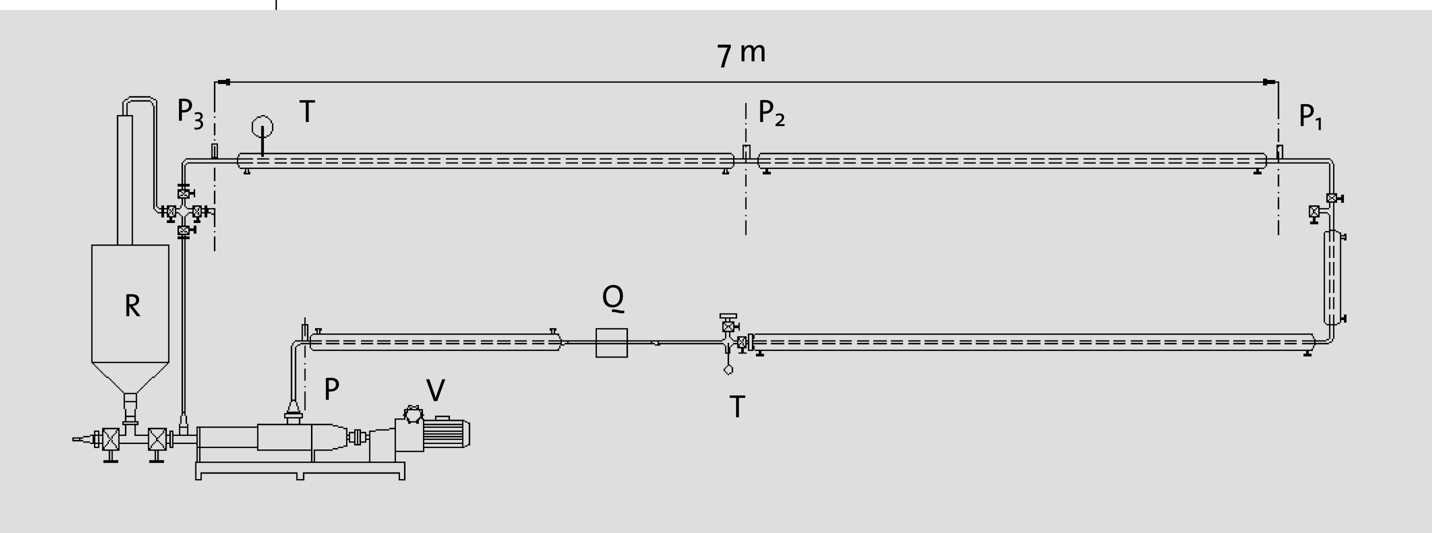
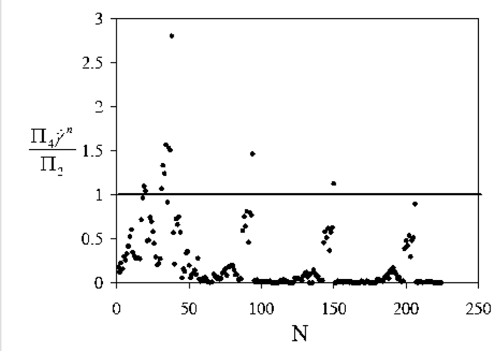


Figure 3 (bottom): The experimental facility. (R) reservoir, (P) volumetric pump and continuous mechanical gear (V), (Q) electromagnetic flow meter, (T) temperature transducers, (P_1, P_2, P_3) pressure transducers.



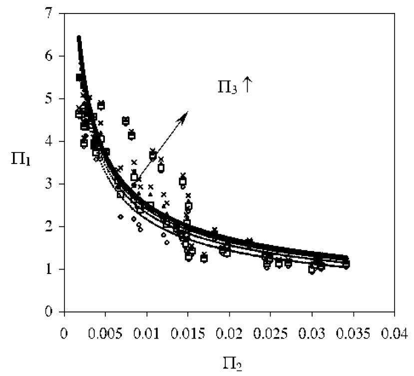


Figure 5 (left above): Plot of data that fulfill the condition expressed by Eq. 6 (N is the cardinal order of the samples: 1 - 48 data at 5 s; 49 - 86 data at 15 s; 97 - 144 data at 30 s; 145 - 192 data at 60 s).

Figure 6 (right above): Dimensionless data as a function of the parameter Π_3 = 78, i.e t = 5 s (◇); Π_3 = 235, i.e. t = 15 s (□); Π_3 = 470, i.e. t = 30 s (▲); Π_3 = 940, i.e. t = 60 s (x).

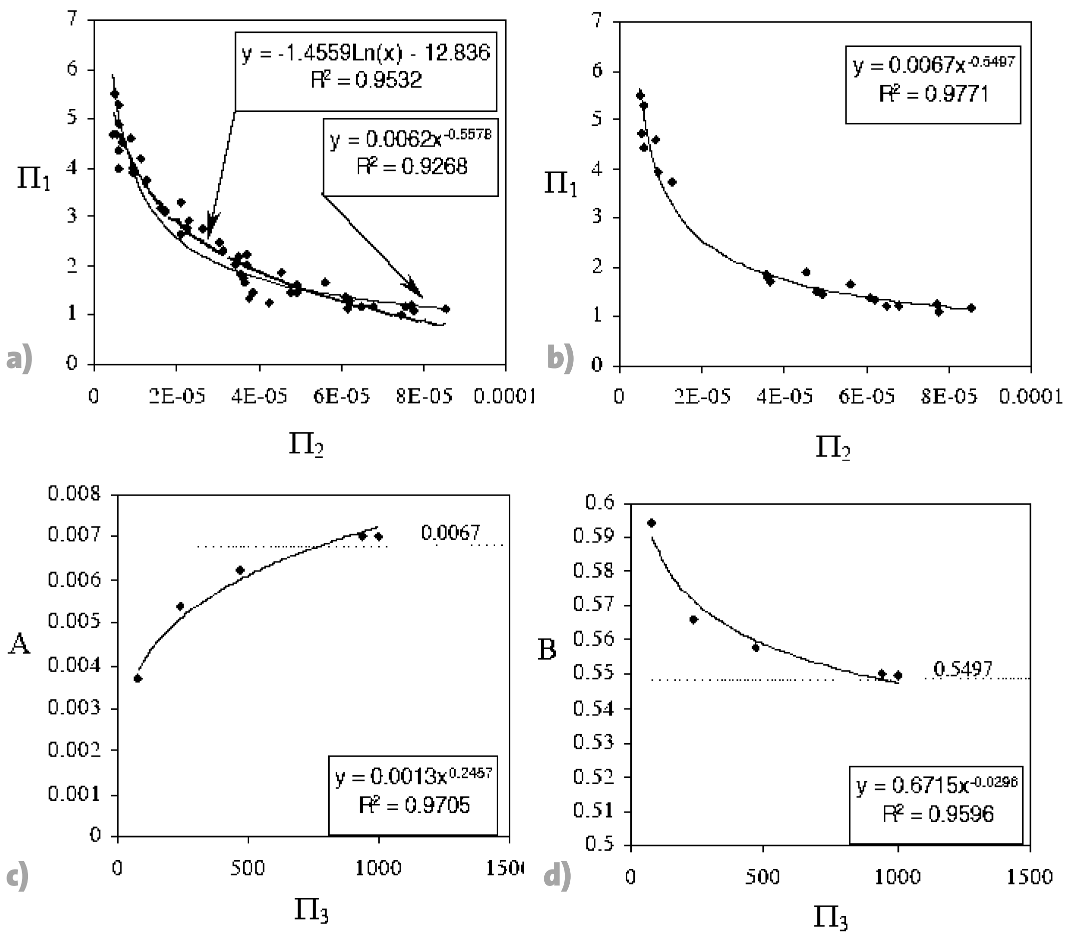


Figure 7: a) Scaling of the data at Π_3 = 470, i.e. t = 30 s and related approximating logarithmic and power laws; b) Scaling of the data at the condition of complete self similarity; c and d) Behaviour of the coefficient of the power law as a function of the parameter Π_3 and related approximating power law curve.

er WIKA with a scale range 0 ÷ 0.1 bar at one end, while the other end was kept at atmospheric pressure. Under incipient motion conditions and in the absence of wall slip effects a simple balance of forces suggests the following link between the wall shear stress τ_w and the yield value τ_o

$$\tau_w = \tau_o = \frac{g\rho D_1 \Delta p}{4L_1} \tag{9}$$

where g is gravity and ρ the fluid density.

Bostwick and rheology

Although the standard Bostwick test B_{30} requires that the position of the front is read at 30 s, here it was sampled at different time instants, i.e. 5, 15, 30 and 60 s. An average of the value from three consecutive identical experiments was then performed to obtain the representative Bostwick value. Data of the fluid front evolution at the different time intervals and for all the samples are summarized in Table I and shown in Fig. 4.

The rheological characterization was done by using an experimental setup based on a temperature controlled rectilinear pipe viscosimeter (Fig. 3), which technical details and performances

Table 2: Values of the constants of the power laws and relative correlation coefficient.

Constant	Value	R^2
a_1	$1.30 \cdot 10^{-3}$	0.97
a_2	$2.46 \cdot 10^{-1}$	0.97
b_1	$6.71 \cdot 10^{-1}$	0.96
b_2	$-2.97 \cdot 10^{-2}$	0.96
A^*	$6.70 \cdot 10^{-3}$	0.97
B^*	$-5.50 \cdot 10^{-1}$	0.97

are reported elsewhere [4, 5]. Here it is worth mentioning that all the measuring instruments (i.e. electromagnetic flowmeter, pressure and temperature transducers) were connected to the LabView data acquisition system. The flow curves (τ, $\dot{\gamma}$) for all the samples were obtained from the measured data of flow rate Q and the corresponding piezometric headloss Δh, by doing the well-known Rabinowtich & Mooney correction (see, for example [3, 4]). For all the fluid samples, the flow curves were fitted to data by using the one dimensional model of Herschel-Bulkley $\tau = \tau_0 + K\dot{\gamma}^n$, whose yield parameter was set equal to that measured.

4.2 RESULTS

First of all, Fig. 4 shows that already after 5 seconds nearly all the samples travelled close to their own final distance. Thus Eq. 6 was verified in practically all the cases, with only some exceptions, i.e. for samples that still possessed an appreciable velocity (see, Fig. 5). However, even these cases finished sooner or later to respect the condition expressed by Eq. 6 and therefore they were also included in the analysis. The dimensionless quantities Π_1 and Π_2 can be plotted in a cartesian diagram, using Π_3 as a parameterizing quantity to identify the position of the front at each dimensionless time steps. The corresponding plot is shown in Fig. 6. Such curves have a well defined shape (Fig. 7a) and are rather closed one another, as expected. It is worth here to stress that this feature reflects the diminishing importance of the quantity Π_3 with time. Accordingly, there is a limiting upper curve that is predicted by the condition of complete self similarity previously described, and to which all the samples should tend at rest. In Fig. 7b such a condition is plot for those samples that at Π_3 = 940 had already stop or by this time possessed a negligible velocity (i.e. this latter considered here to be less than 2 mm/min, for practical purposes). From the shape of the curve it turns out that a reasonably hypothesis to express Eq. 8 is a power law relationship

$$\Pi_1 = A^* \Pi_2^{B^*} \tag{10}$$

with constant parameters A^* = 0.0067 and B^* = -0.5497.

For applicative purposes, it would be interesting to find out a relationship able to account for the weak transitory in the data, i.e. where Eq. 6 holds. A way to do this is to parameterize the curves of Fig. 6 as a function of Π_3. Although the data on such diagram would seem to require a more complex relationship (i.e. for example a logarithmic one), for sake of simplicity a convenient extension of the former Eq. 10 was still adopted here. Non stationarity, was therefore included by making the parameters A and B not constant, but functions of the variable Π_3,

$$\Pi_1 = A(\Pi_3)\Pi_2^{B(\Pi_3)} \tag{11}$$

Figures 7c, d show how the coefficients A and B should actually varying. It is remarkable to notice that such coefficients asymptotically tend to the values that are valid for the case of complete self similarity, as expected. A not too much fortunate, but for sake of simplicity, convenient choice is to describe such a behaviour by adopting again a simple power law relationship

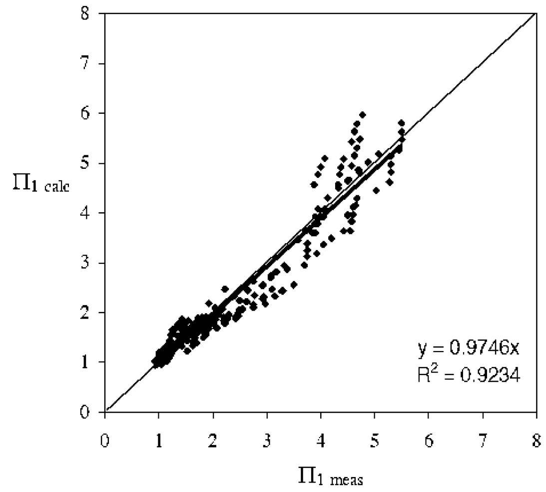
$$A = a_1 \Pi_3^{a_2} \tag{12}$$

$$B = b_1 \Pi_3^{b_2} \tag{13}$$

By means of a common fitting procedure, the final form of the correlation becomes

$$\Pi_1 = a_1 \Pi_3^{a_2} \Pi_2^{b_1 \Pi_3^{b_2}} \tag{14}$$

whose parameters values are listed in Table 2, whilst the corresponding diagram is shown in Fig. 8. Such a correlation relates therefore the yield stress and the front length evolution at different instants of time into a unique curve. As a



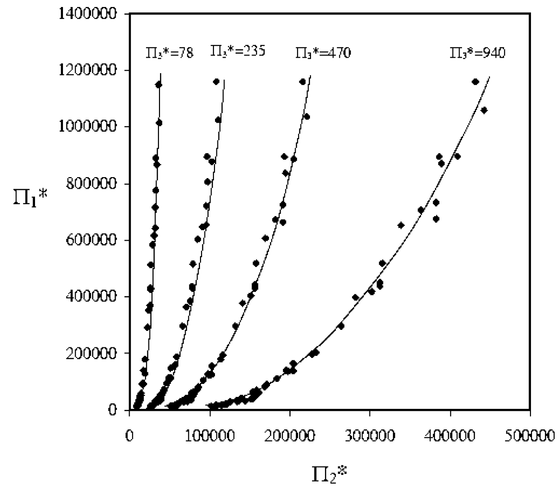


Figure 8 (left): Global scaling. Predicted vs. measured data.

Figure 9 (right): Scaling of the data in the new system of variables.

consequence it allows to estimate the Bostwick consistency B_{30} starting from the knowledge of the yield stress τ_0 and the fluid density ρ only. The step back, i.e. to assess the yield stress from the consistency B_{30}, in principle should be also possible by averaging the results coming from different measures of front evolutions. A final comment is now reserved in order to show how the dimensionless groups introduced here can be used to transforming and making thus useful the diagram of Fig. 4. To this end, a new plot can be obtained after a convenient change of variables is introduced. By defining, for example

$$\Pi_{1*} = \frac{\Pi_1}{\Pi_2} = \frac{LgM}{\tau_o H^3} \quad (15)$$

$$\Pi_{2*} = \frac{\Pi_3}{\Pi_2^{1/2}} = \frac{tgM^{1/2}}{\tau_o^{1/2} H^{3/2}} \quad (16)$$

$$\Pi_{3*} = \Pi_3 \quad (17)$$

the quantities τ_0 and t are introduced into the dimensionless variables Π_1 and Π_2. The major effect of such a transformation of coordinates is to distorting the curve on the cartesian plane (Π_1, Π_2), but still preserving the possibility of parameterizing all the curves at Π_{3*} = *cost*. The related effect on data is that now they scale as shown in Fig. 9, and such curves can potentially be used from a graphics point of view.

5 DISCUSSION AND CONCLUSIONS

The results of this work are of course subjected to measurement errors and simplifications, which all contributed to give the discrepancies that are still evident in the proposed correlations. In particular, there are effects which were either too complex or very difficult to describe that were therefore not included in the analysis. Examples are the width of the flowing lane and the role of the roughness of the flow lane. While the effects due to the first variable could be easily explored, for the second one the point is much more delicate. In facts, as far as B_{30} is concerned, the difference using a rough plane can be as much as 20 - 30% of the measure obtained with a smooth one (Private communication by a Company producing Bostwick consistometers, name omitted). In particular, these errors would seem to be due to the separation of the solid (fibers and other solid components) and liquid (i.e. the serum) phases of the product. In turn this would give rise to density variation and reduction of the internal lubrication. As a consequence, an increasing of the yield stress can be expected, eventually influencing the apparent viscosity at low shear rates.

In order to proceed toward a better standardization of the Bostwick test, which would simplify the comparison of a wider source of data, the above mentioned effects should be first of all better investigated or at least better documented. Thus, users could perhaps be provided of an official library of correcting factors. Doing such a step, however, requires greater efforts from both the experimental and analytical viewpoints. Theoretical and experimental approaches are therefore welcome, especially if based on physically consistent assumptions that could improve the knowledge significantly beyond avoiding confusion. For instance, the rheological nature of the fluid being tested should be considered since the beginning. This first step missing would bring inexorably to the aforementioned Eolkin's paradox. Aiming at the use of the Bostwick measure in a more wider sense therefore implies the necessity of taking the physics that lies behind the phenomenon under more consideration (for instance following [14, 15]).

This would avoid to propagate and turn into confusion the 'ignorance' that the Bostwick measure implicitly introduces in the results. Future steps should therefore aim at introducing the nonlinear role of yield stress in the front evolution following the traces already suggested by [18, 19, 20, 23]. Although the numerical work by Mei and Yuhi [19] already considered the limiting case of rectangular channel, some additional efforts could still help to complete the studies already done by [14, 15]. In particular, this could be done by extending the results in [19] to a horizontal bed and by comparing both numerical and experimental results.

In conclusion, although this work did not lead to a physical model, it allowed to critically analyze the role of the involved quantities, and to obtain novel results of broad scientific interest. Only yield stress fluids were deliberately focused on here, which are relevant to several fields of engineering. Indeed, beyond food engineering - in which this correlation can be useful to control the food process quality in modern industrial plants - many other processes of geophysics [23] and hydraulics [24, 25] can potentially use such an analysis. As an example, the Bostwick test offers the possibility to study the dynamics succeeding a rapid initial transitory. This process shows many similarities with the problems of 'dam-break' of very viscous fluids (see, for example [25]), but at a smaller scale.

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